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Harmonic Vs. transient analysis of a vehicle

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Abstract

Harmonic (frequency response) analysis is often carried out when the input loading is simple harmonic, for instance, when the vehicle is moving on a speed breaker on the road. In the harmonic analysis, the transient vibrations are ignored, resulting in large errors in the calculations of deformations and stresses. In this work, vibration analyses of a vehicle are performed by both harmonic and transient dynamic approaches for the vehicle moving on a speed breaker, which gives a harmonic loading to the vehicle. It is seen that there is a significant difference in the predicted life. This causes large errors in the predicted fatigue life and comfort, especially at the higher excitation frequencies.

Keywords–*Vibration, Transient, Harmonic, total response, steady-state*

I. INTRODUCTION

The response analysis, also called harmonic analysis, is widely used for analyzing structures subject to harmonic loads like washing machines, motors, automobiles. The design validation of a vehicle moving on a speed breaker is often performed using frequency response analysis. The plot of an undamped **natural** vibration of a single degree of freedom system as seen from the displacement Vs. The time plot is harmonic. But the displacement Vs. The time plot of an undamped single degree of freedom system subject to harmonic loading is NOT harmonic. A popular misconception among the structural analysts is to believe that it is harmonic. The assumption that the motion is necessarily harmonic leads to harmonic analysis.

The equation of motion for a single degree of freedom spring-mass vibrator with spring stiffness K and mass M with damping coefficient c, and displacement of mass, u, is [1]:

$$M\ddot{u} + C\dot{u} + Ku = f(t) \quad ...(1)$$

Where f(t) represents the time-dependent force acting on the mass. $\ddot{u}$and$\dot{u}$ are acceleration and velocity, respectively.

Here, it is assumed that the system is structurally linear with small displacements and elastic material.

Sinusoidal variation of motion is assumed for the harmonic loading giving rise to the equations [2]

$F(t) = F_0 \sin(\omega.t)$ and

$u(t) = U_0 \sin(\omega.t)$

where F_0 and U_0 are the force and displacement amplitudes, respectively, and ω and $\omega_n$are the excitation frequency and the natural frequency, respectively.

$F0 = mr\omega^2$

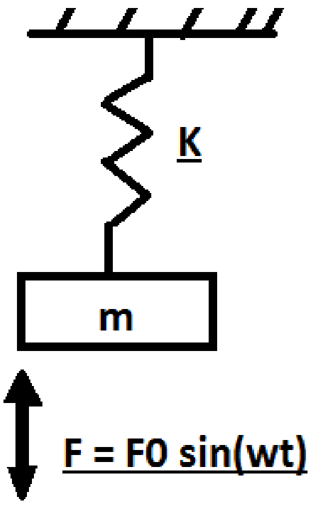

Figure 1 Single DOF vibrator with harmonic loading

Most of the metallic structures in the industry are approximated by a damping ratio of 2% [3-8], which is lightly damped and can be approximated as undamped vibration for the initial period of vibration. The displacement of the mass at the time, t is [2] given by:

$$u = \frac{(F_0 / K)(\sin(\omega.t)}{1 - r^2} \quad ...(2)$$

Where,

$$r = \frac{\omega}{\omega_n}$$

The above relation is valid only for the steady-state part of the vibration, and in this expression, the transient vibrations are ignored. Often the total response is approximated to be equal to the steady-state response, and equation (4) is assumed to give the total vibratory response of the system.

The analytical formula incorporating the transient vibrations as well as the steady-state vibrations is

$$u = \frac{(F_0 / K)\{(\sin(\omega.t) - r.\sin(\omega_n.t)\}}{1 - r^2} \quad ...(3)$$

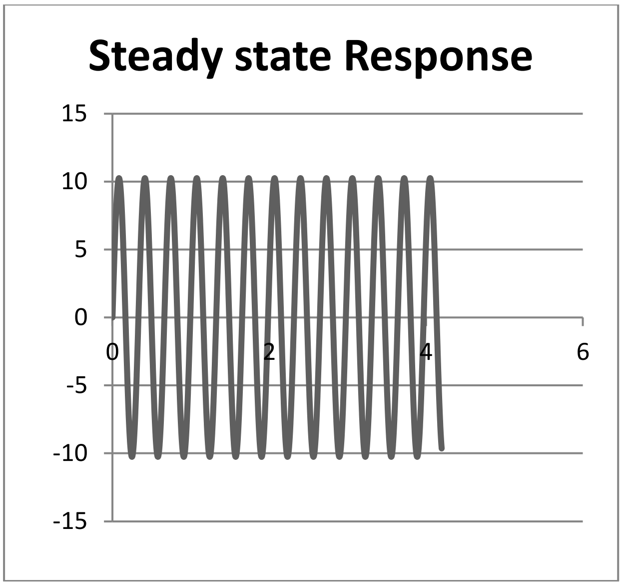

Figure 2 Steady-state vibrations

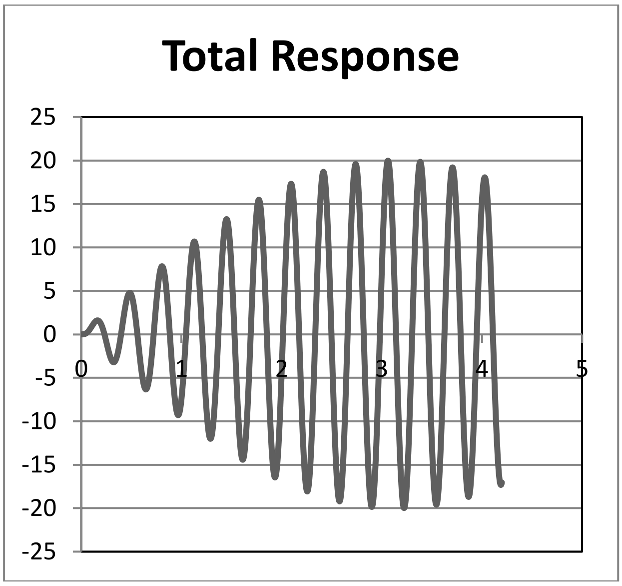

Figure 3 Transient + steady state vibrations

The harmonic response considers only the steady-state response and ignores the transient response, which is essential initially.

From equation 3, the maximum amplitude reached by the single degree of freedom system is [9]

u max = u steady state. (1+r)

it must be noted that the factor 1+r denotes that the actual induced displacements and hence the induced stresses in the initial stage of vibration are much larger than that indicated by the harmonic analysis, which takes into account only the steady-state part of vibrations.

II. ROLE OF TRANSIENT VIBRATIONS IN VARIATION OF AMPLITUDE

If the harmonic response is assumed, then the maximum amplitude would be reached in the first cycle itself, in which case response cannot be calculated for undamped resonance since it would mean infinite amplitude in a finite interval of time, but even in a damped system, the response would be sudden reaching the maximum amplitude in the first vibration cycle. However, in reality, the amplitude increases gradually, implying that the total response is not just a steady-state response, even for damped systems. A simplistic assumption that can be made here is that since undamped systems do not exist in reality, the discussion on undamped resonance is useless. That would be incorrect because the undamped vibrations increase their amplitude steadily just as damped vibration amplitude tends towards the steady-state amplitude at a gradually decreasing rate. The study of undamped vibrations helps the vibration students understand the phenomenon of resonance without considering the complication of damping.

It can be seen that while the steady-state amplitude is constant, the total amplitude varies with time. The steady-state vibrations continue as though the transient vibrations are absent and vice versa. The total response is the sum of the transient and steady-state vibrations. The transient and steady-state vibrations are initially in the opposite phase, and hence, the total response is small initially. As the vibrations continue, the relative phase angle change, and the amplitude increases and reaches maximum amplitude when the phase difference is minimum. Again after some time, the phase difference changes resulting in a change in amplitude. In general, the amplitude is highest when the phase difference is least, and the amplitude is least when the phase difference is at its maximum.

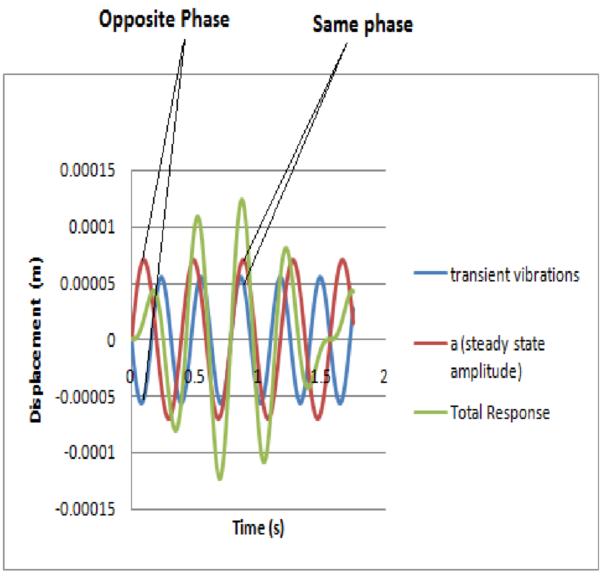

Figure 4 Transient, steady-state and total response

Total response = Steady state response + transient response

At the same phase,

The amplitude of total response = Amplitude of steady-state response + amplitude of the transient response.

At the opposite phase,

The amplitude of total response = Amplitude of steady-state response - amplitude of the transient response.

It may also be noted that the steady-state vibrations do not absorb energy from or give out energy to the exciting agency per cycle. However, transient vibrations absorb at some intervals of time and give out energy to the exciting external agency at other intervals of time, even in undamped systems.

III. CASE STUDY: AN AUTOMOBILE MOVING ON A SPEED BREAKER IN THE ROAD

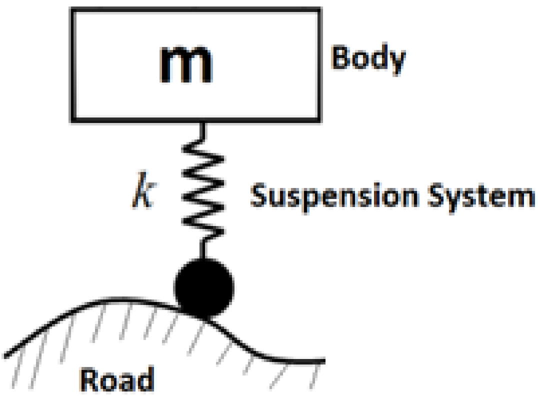

Figure 5 Vehicle on a speed-breaker

The vehicle is in contact with the ground, causing the boundary conditions to be problematic [10]. However, often, a simplified analysis assuming fixed boundary conditions can be accurate. The automobile is idealized as a single degree of freedom system. The suspension system has a stiffness of 2.56 MN/m; the mass of the body is 200 Kg. the natural frequency of the system is 18 Hz. The wheel moves over the speed breaker in 0.066 secs. Thus the frequency of excitation is 15 Hz.

The vehicle moves with a constant speed v. at any given moment, the vertical displacement is

d = d0. Sinωt

where,

ω = frequency of road excitation = 2πf = 94.24 rad/s; t is the time

And d0 is the height of the speed breaker = 0.1m.

Vertical Velocity = d0.ω.cosωt

The acceleration = -ω^2.d0.sinωt

The exciting force acting on the wheel, f = ω^2.d0.sinωt = F0.sinωt

Where, F0 = ω^2.d0= 888.26 N

If the vehicle receives a continuous road excitation of the above nature, the maximum displacement the body will experience as per the steady-state formula

$$u_{\text{steady state}} = \frac{(F_0/K)}{1-r^2} = 1.1 \text{ mm}$$

But if the transient vibrations are included, maximum displacement, u = 2.077 mm

It may be noted that a ride that is predicted to be comfortable as per the steady-state formula may not satisfy the comfort requirements of the ride since the actual displacement can be more extensive.

The calculation using the steady-state formula can lead to fatigue design failure of the vehicle too. The measurement of strain during vibration is difficult; hence the automotive engineers depend upon finite element analysis to calculate fatigue life. The fatigue life is carried out based on the stresses determined by the harmonic analysis [11,12] using the stress-life method [13]. In this method, the predicted stress life is found based on the predicted stress levels.

The fatigue lives N1 and N2 at two different stress amplitude levels, s1 and s2 are related by the formula [14]:

$$\frac{N1}{N2} = \left(\frac{s2}{s1}\right)^{1/b}$$

Where,

b is the fatigue ductility exponent = 0.1 for steel

If s1 is the stress amplitude determined by the harmonic analysis and s2 is the actual stress amplitude determined by the transient dynamic analysis. Then,

$$\frac{s2}{s1} = 1 + r$$

and

$$N2 = \frac{N1}{(1+r)^{1/b}}$$

Since b =0.1

$$N2 = \frac{N1}{(1+r)^{10}}$$

IV effect of damping

The damping causes the transient vibrations to be limited to the first few cycles of vibration. If the effect of transient vibrations is limited to n% of the time,

The total damage is

D =

$$\frac{N.n}{N2} + \left(1 - \frac{n}{100}\right)\frac{N}{N1}$$

$$= N.\left(\frac{n}{100.N2} + \left(1 - \frac{n}{100}\right).\frac{1}{N1}\right)$$

Where,

N is the total number of vibration cycles the body can withstand before the crack formation.

Fatigue failure occurs when D =1

⇨ Failure occurs when

$$N.\left(\frac{n}{100.N2} + \left(1 - \frac{n}{100}\right).\frac{1}{N1}\right) = 1$$

⇨ N =

$$\frac{1}{\left(\frac{n}{100.N2} + \left(1 - \frac{n}{100}\right).\frac{1}{N1}\right)}$$

$$N2 = \frac{N1}{(1+r)^{10}}$$

But

Therefore,

N =

$$\frac{1}{\left(\frac{n.(1+r)^{10}}{100.N1} + (1-\frac{n}{100}).\frac{1}{N1}\right)}$$

=

$$\frac{N1}{\frac{n.(1+r)^{10}}{100} + 1 - \frac{n}{100}}$$

The systems are usually designed for 3million cycles. Taking N1 = 3x 10^6 cycles, we find that N decreases with an increase in r and n.

For the given vehicle, the N Vs. n plot is shown in fig.6

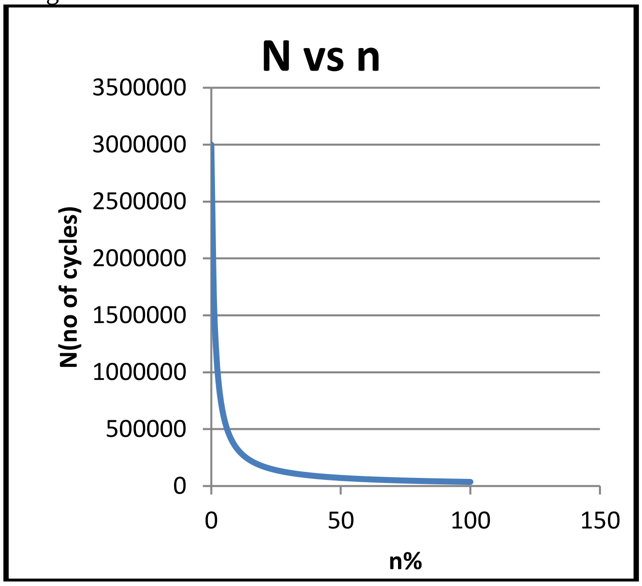

Figure 6 Fatigue life, NVs percentage of High amplitude content, n

IV. DISCUSSION

The amount of displacement, as predicted by the harmonic analysis, is lesser than the actual displacement as predicted by the transient dynamic analysis. Transient vibrations cause the changes in amplitude. The expressions derived show that fatigue life decreases rapidly with an increase in the high amplitude content.

V. CONCLUSION

The displacements and stresses, as predicted by the harmonic analysis, are lesser than the actual in the initial phase of vibration. Damping may decrease the high amplitude content of vibration. However, even a small amount of such vibration can significantly affect the vehicle ride comfort and the durability of the vehicle parts, especially in the presence of high-frequency excitations, which cause large design errors. It is recommended that the transient analysis be conducted even for the machine parts subject to harmonic loads.

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