# Analysis of Transmission Lines by Double Rohit Transform 

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#### Abstract

The term transferral or transmission or transference line points to a specialized wire manufactured from copper or aluminum laid out with excellent dielectric or non-conductor and used for the transferral of electrical energy. Generally, a transferral line has a resistor put up by the two wires taken together, an inductor, a shunt conductance and a capacitor. The four quantities form the principal criterion of the transferral lines, and they turn on the class and assembly of transferral lines. Traditionally, the wave equations of transferral lines are examined by the Fourier transform or Laplace Transform. These transform procedures are very opportune for examining typical wave equations of electric transferral lines. The $R T$ is the latest integral transform technique that has been tried to examine the boundary value differential equations standing up in separate engineering scopes. This paper disburses the solution of wave equations of transferral lines having negligible losses to earth or deck through dielectric by the double RT. The double RT will transpire to be a very positive tool for examining the wave equations of transferral lines.


Keywords - Transferral line, Double rohit transform, Electrical insulation.

## 1. Introduction

The term transferral or transmission or transference line points to a specialized wire manufactured from copper or aluminum laid out with excellent dielectric or non-conductor and used for the transferral of electrical energy. Generally, a transferral line has a resistor put up by the two wires taken together, an inductor, a shunt conductance and a capacitor. The four quantities form the principal criterion of the transferral lines, and they turn on the class and assembly of transferral lines [1]. The transmission line is a very frequently used distributed circuit: a set of wires. Woefully, a set of wires used to put in a time-varying voltage is not ideal when the wavelength of the signal appealed is comparable to the length of the wires. A brief analysis of the transmission line is as follows. A small section of a transmission line is modelled as a combination of a series resistance, a series inductance, a shunt conductance and a shunt capacitance. A transmission line is made up of a cascade of a few of these small sections. The resistance, inductance, capacitance and conductance values are proportional to the length of the transmission line section. They are generally represented with the help of per-unit-length quantities.

Transmission line is the paramount part of the power system. The need for power and its allegiance has increased exponentially over the modern era, and the main role of a transmission line is to transmit electric power from the source
area to the distribution network. The explosion between limited production and a tremendous claim has increased the focus on reducing power losses. Losses like transmission loss and physical losses to various technical losses [2, 3].

A transmission-line section is shown in the figure below.


This paper examines the wave equations of transferral lines with negligible losses negligible losses to the earth or deck through dielectric via the double RT. Taking into account the semi-infinite electric transferral lines with a continual voltage $\mathrm{V}_{0}$ appealed at its sending end $(\mathrm{z}=0)$ at t $=0$. If $V(z, t)$ is the voltage and $I(z, t)$ is the electric current at any point ( $\mathrm{z}, \mathrm{t}$ ), then the equations announcing the progress of electric current and electric potential on lossy transferral lines [4] are given by

$$
\begin{equation*}
-\frac{\partial V(z, t)}{\partial z}=R I(z, t)+L \frac{\partial I(z, t)}{\partial t} \tag{1}
\end{equation*}
$$

$$
\begin{equation*}
\text { and }-\frac{\partial I(z, t)}{\partial z}=G V(z, t)+C \frac{\partial V(z, t)}{\partial t} \tag{2}
\end{equation*}
$$

Differentiating (1) with regard to z and differentiating (2) with regard to $t$, and simplifying, we find

$$
\begin{equation*}
\frac{\partial^{2} V(z, t)}{\partial z^{2}}=\left\{C L \frac{\partial^{2}}{\partial t^{2}}+(\mathrm{GL}+\mathrm{CR}) \frac{\partial}{\partial t}+G R\right\} V(z, t) \tag{3}
\end{equation*}
$$

Differentiating (1) with regard to $t$ and differentiating (2) with regard to z and simplifying, we find

$$
\begin{equation*}
\frac{\partial^{2} I(z, t)}{\partial z^{2}}=\left\{C L \frac{\partial^{2}}{\partial t^{2}}+(\mathrm{GL}+\mathrm{CR}) \frac{\partial}{\partial t}+G R\right\} I(z, t) \tag{4}
\end{equation*}
$$

These equations constitute the general wave equations for lossy electrical transferral lines.

For negligible losses to the ground on an electric transferral line, we put $G=0$ and $L=0$, because these parameters are in charge of leakages on the electric transferral lines [3]. Therefore, we can rewrite (3) and (4) as:

$$
\begin{align*}
\frac{\partial V(z, t)}{\partial z} & =-R I(z, t)  \tag{5}\\
\text { and } \quad \frac{\partial I(z, t)}{\partial z} & =-C \frac{\partial V(z, t)}{\partial t} \tag{6}
\end{align*}
$$

Here,
$V(z, 0)=0, V(0, t)=V_{0}, I(z, 0)=0 \& V(z, t)$ is finite at each point $(\mathrm{z}, \mathrm{t})$.
We have to work out (5) and (6) by the Double RT.
The RT [5-6] of $g(z), \mathrm{z} \geq 0$ is denoted by $G(q)$ and is given by $G(q)=q^{3} \int_{0}^{\infty} e^{-q y} g(z) d z$, provided the integral is convergent.

The complex inversion formula for RT is given by $g(y)=\frac{1}{2 \pi i} \int_{b-i \infty}^{b+i \infty} \frac{e^{r y}}{r^{3}} G(r) d r=$ sum of residues of $e^{r y} \frac{G(r)}{r^{3}}$ at the poles of $\frac{G(r)}{r^{3}}$.

Proof:
The RT of $g(y), y \geq 0$ is

$$
G(r)=r^{3} \int_{0}^{\infty} e^{-r t} g(t) d t
$$

Multiply both sides by $\frac{e^{r y}}{r^{3}}$, we have

$$
\frac{e^{r y}}{r^{3}} G(r)=e^{r y} \int_{0}^{\infty} e^{-r t} g(t) d t
$$

Integrate w.r.t. r between the limits $\mathrm{b}-\mathrm{ih}$ and $\mathrm{b}+\mathrm{ih}$; we have

$$
\int_{b-i h}^{b+i h} \frac{e^{r y}}{r^{3}} G(r) d r=\int_{b-i h}^{b+i h} e^{r y} d r \int_{0}^{\infty} e^{-r t} g(t) d t
$$

Put $\mathrm{r}=b-i p, d r=-i d p$, we have

$$
\int_{b-i h}^{b+i h} \frac{e^{r y}}{r^{3}} G(r) d r=-\mathrm{i} \int_{h}^{-h} e^{(b-i p) y} \int_{0}^{\infty} e^{-(b-i p) t} g(t) d t d p
$$

$$
\begin{equation*}
\int_{b-i h}^{b+i h} \frac{e^{r y}}{r^{3}} G(r) d r=\mathrm{i} e^{b y} \int_{-h}^{h} e^{-i p y} \int_{0}^{\infty} e^{-b t} e^{i p t} g(t) d t d p \tag{i}
\end{equation*}
$$

Let us define a new function: $f(y)$ $=\left\{\begin{array}{c}e^{-b y} g(y) \text { when } \mathrm{y} \geq 0 \\ 0 \text { wnen } \mathrm{y}<0\end{array}\right.$

The Fourier complex integral of the function $f(y)$ is given by

$$
\begin{align*}
& \mathrm{f}(\mathrm{y})=\frac{1}{2 \pi} \int_{-\infty}^{\infty} e^{-i p y} \int_{-\infty}^{\infty} f(t) e^{i p t} d t d p \\
& e^{-b y} g(y)=\frac{1}{2 \pi} \int_{-\infty}^{\infty} e^{-i p y} \int_{0}^{\infty} e^{-b t} g(t) e^{i p t} d t d p \\
& 2 \pi e^{-b y} g(y)=\int_{-\infty}^{\infty} e^{-i p y} \int_{0}^{\infty} e^{-b t} g(t) e^{i p t} d t d p \tag{ii}
\end{align*}
$$

In the limiting case when $\mathrm{h} \rightarrow \infty$, equation ( $i$ ) becomes,

$$
\begin{equation*}
\int_{b-i \infty}^{b+i \infty} \frac{e^{r y}}{r^{3}} G(r) d r=\mathrm{i} e^{b y} \int_{-\infty}^{\infty} e^{-i p y} \int_{0}^{\infty} e^{-b t} e^{i p t} g(t) d t d p \tag{iii}
\end{equation*}
$$

Using (ii) and (iii), we have

$$
\begin{aligned}
& \int_{\substack{b-i \infty \\
b+i \infty}}^{\substack{ \\
b+i \infty}} \begin{array}{l}
\int_{b-i \infty}^{r y} \\
e^{r y} \\
r^{3}
\end{array}(r) d r=\mathrm{i} e^{b y} 2 \pi e^{-b y} g(y) d r=2 \pi i g(y)
\end{aligned}
$$

$$
\begin{equation*}
g(y)=\frac{1}{2 \pi i} \int_{b-i \infty}^{b+i \infty} \frac{e^{r y}}{r^{3}} G(r) d r \tag{iv}
\end{equation*}
$$

### 1.1. This is known as the inversion formula for the Rohit transform

To obtain $g(y)$ the integration is performed along a line $A B$ parallel to the imaginary axis in the complex plane such that all the singularities of $\frac{G(r)}{r^{3}}$ lie to its left. Contour C includes line AB and semicircle BDA .


From equation (iv),
$g(y)=\frac{1}{2 \pi i} \int_{A B} \frac{e^{r y}}{r^{3}} G(r) d r$
$g(y)=\frac{1}{2 \pi i} \int_{c} \frac{e^{r y}}{r^{3}} G(r) d r-\frac{1}{2 \pi i} \int_{B D A} \frac{e^{r y}}{r^{3}} G(r) d r$
The integration over BDA tends to zero as $h \rightarrow \infty$. therefore,

$$
\begin{equation*}
g(y)=\lim _{\mathrm{h} \rightarrow \infty} \frac{1}{2 \pi i} \int_{c} \frac{e^{r y}}{r^{3}} G(r) d r \tag{v}
\end{equation*}
$$

Hence, $g(y)$ sum of residues of $\frac{e^{r y}}{r^{3}} G(r)$ at the poles of $\frac{G(r)}{r^{3}}$.

The double RT of $g(z, t)$, a function of two variables $z>$ $0, \mathrm{t}>0$, is defined as
$R_{z} R_{t}[g(z, t)]=G(q, r)=$
$q^{3} r^{3} \int_{0}^{\infty} e^{-q z} \int_{0}^{\infty} e^{-r t} g(z, t) d z d t$,
where $q$ and r are complex numbers.
The double RT of the first-order partial derivative [7-9] of $g(z, t)$ is defined as:
$R_{z} R_{t}\left[\frac{\partial g(z, t)}{\partial z}\right]=q G(q, r)-q^{3} G(0, r)$,
and
$R_{z} R_{t}\left[\frac{\partial g(z, t)}{\partial t}\right]=\mathrm{r} G(q, \mathrm{r})-\mathrm{r}^{3} G(q, 0)$.
The double RT of the second-order derivatives [10, 11] is defined as:
$R_{z} R_{t}\left[\frac{\partial^{2} g(z, t)}{\partial z^{2}}\right]=q^{2} G(q, \mathrm{r})-q^{4} G(0, \mathrm{r})-q^{3} \frac{\partial G(0, \mathrm{r})}{\partial z}$,
and
$R_{z} R_{t}\left[\frac{\partial^{2} g(z, t)}{\partial t^{2}}\right]=\mathrm{r}^{2} G(q, \mathrm{r})-\mathrm{r}^{4} G(q, 0)-\mathrm{r}^{3} \frac{\partial G(q, 0)}{\partial t}$.
The RT of some basic functions $[12,13]$ is given as

$$
\begin{aligned}
& >R\left\{t^{n}\right\}=\frac{n!}{s^{n-2}}, \text { where } n=0,1,2, \ldots \\
& >R\left\{e^{e t}\right\}=\frac{s^{3}}{s-e}, s>e, \text { where e is some constant. } \\
& >R\{\text { sinet }\}=\frac{e s^{3}}{s^{2}+e^{2}}, s>0 \\
& >R\{\text { coset }\}=\frac{s^{4}}{s^{2}+e^{2}}, s>0
\end{aligned}
$$

## 2. Materials and Methods

### 2.1. Highlights

- This paper focuses on the administration of Rohit transform, also written as RT, an upto the minute integral transform technique.
- A double RT approach is proposed for handing out the solution of wave equations of transferral or transmission or transference lines having negligible losses through dielectric to the earth or deck.
- Highly precise and accurate results were obtained.
- Focusing on any research effort concerning literature review is the very paramount task because it builds up ideas that can evolve quickly. This paper dispenses a new methodology for solving transmission line equations. It shows beyond doubt that the integral transform RT is a potent mathematical tool to deal with such problems.

Now, differentiating (5) with regard to z and employing (6), we find

$$
\begin{equation*}
\frac{\partial^{2} V(z, t)}{\partial z^{2}}=\operatorname{CR} \frac{\partial V(z, t)}{\partial t} \tag{7}
\end{equation*}
$$

Taking double RT of (7), we have

$$
\begin{align*}
& R_{z} R_{t}\left\{\frac{\partial^{2} V(z, t)}{\partial z^{2}}\right\}=\operatorname{CR} R_{z} R_{t}\left\{\frac{\partial V(z, t)}{\partial t}\right\} \\
& \text { Or } \begin{aligned}
r^{2} \overline{\mathrm{~V}}(\mathrm{r}, \mathrm{~s})-r^{4} \overline{\mathrm{~V}}(0, \mathrm{~s}) & -r^{3} \frac{\partial}{\partial z}[\overline{\mathrm{~V}}(0, \mathrm{~s})] \\
& =\operatorname{CR}\left[s \overline{\mathrm{~V}}(\mathrm{r}, \mathrm{~s})-s^{3} \overline{\mathrm{~V}}(\mathrm{r}, 0)\right]
\end{aligned}
\end{align*}
$$

As $\overline{\mathrm{V}}(0, \mathrm{~s})=s^{2} \mathrm{~V}_{0}, \overline{\mathrm{~V}}(\mathrm{r}, 0)=0$ and put $\frac{\partial}{\partial z}[\overline{\mathrm{~V}}(0, \mathrm{~s})]=$ $P(s)$, therefore, (8) gives
$\mathrm{r}^{2} \overline{\mathrm{~V}}(\mathrm{r}, \mathrm{s})-\mathrm{r}^{4} s^{2} \mathrm{~V}_{0}-\mathrm{r}^{3} P(\mathrm{~s})=C R[s \overline{\mathrm{~V}}(\mathrm{r}, \mathrm{s})]$
Rearranging the equation, we get

$$
\begin{equation*}
\overline{\mathrm{V}}(\mathrm{r}, \mathrm{~s})=\frac{\mathrm{r}^{4} s^{2} \mathrm{v}_{0}}{\mathrm{r}^{2}-\mathrm{CRs}}+\frac{\mathrm{r}^{3} P(s)}{\mathrm{r}^{2}-\mathrm{CRs}} \tag{9}
\end{equation*}
$$

Taking double inverse RT of (9) with regard to $\mathrm{t} \& \mathrm{~s}$, we find
$R_{s}^{-1} R_{\mathrm{r}}^{-1}\{\overline{\mathrm{~V}}(\mathrm{r}, \mathrm{s})\}=R_{s}^{-1} R_{\mathrm{r}}^{-1}\left\{\frac{\mathrm{r}^{4} s^{2} \mathrm{~V}_{0}}{\mathrm{r}^{2}-\mathrm{CRs}}+\frac{\mathrm{r}^{3} P(s)}{\mathrm{r}^{2}-\mathrm{CRs}}\right\}$
Or
$R_{S}^{-1}\{\overline{\mathrm{~V}}(\mathrm{z}, \mathrm{s})\}=R_{S}^{-1}\left\{s^{2} \mathrm{~V}_{0} \cosh \sqrt{\mathrm{CRs}} Z\right.$ $\left.+\frac{P(s)}{\sqrt{\mathrm{RCs}}} \sinh \sqrt{\mathrm{CRs}} Z\right\}$
Or
$R_{S}^{-1}\{\overline{\mathrm{~V}}(\mathrm{z}, \mathrm{s})\}=R_{S}^{-1}\left\{s^{2} \mathrm{~V}_{0} \frac{e^{\sqrt{\mathrm{CRs}} z}+e^{-\sqrt{\mathrm{CRs}} z}}{2}\right.$
$\left.+\frac{P(s)}{\sqrt{\mathrm{CRs}}} \frac{e^{\sqrt{\mathrm{CRs}} z}-e^{-\sqrt{\mathrm{CRs}} z}}{2}\right\}$
Or

$$
\begin{gather*}
R_{s}^{-1}\{\overline{\mathrm{~V}}(\mathrm{z}, \mathrm{~s})\}=R_{s}^{-1}\left\{\left(\frac{s^{2} \mathrm{~V}_{0}}{2}+\frac{P(s)}{2 \sqrt{\mathrm{CRs}}}\right) e^{\sqrt{\mathrm{CRs}} z}+\left(\frac{s^{2} \mathrm{~V}_{0}}{2}-\right.\right. \\
\left.\left.\frac{P(s)}{2 \sqrt{\mathrm{CRs}}}\right) e^{-\sqrt{\mathrm{CRs}} z}\right\} \tag{10}
\end{gather*}
$$

Since $\overline{\mathrm{V}}(\mathrm{z}, \mathrm{s})$ is finite as $z \rightarrow \infty$, therefore,
on putting $\frac{s^{2} \mathrm{~V}_{0}}{2}+\frac{P(s)}{2 \sqrt{\mathrm{RCs}}}=0$
Or

$$
\begin{gather*}
\mathrm{P}(\mathrm{~s})=-s^{2} \mathrm{~V}_{0} \sqrt{\mathrm{CRs}} \text { in }(10), \text { we get } R_{s}^{-1}\{\overline{\mathrm{~V}}(\mathrm{z}, \mathrm{~s})\}= \\
R_{s}^{-1}\left\{s^{2} \mathrm{~V}_{0} e^{-\sqrt{\operatorname{RCs}} z}\right\} \tag{11}
\end{gather*}
$$

Since $\operatorname{erf}\left(\frac{a}{2 \sqrt{\mathrm{z}}}\right)=\frac{2}{\sqrt{\pi}} \int_{0}^{\frac{a}{2 \sqrt{z}}} e^{-x^{2}} \mathrm{dx}, \quad[14, \quad 15] \quad$ therefore, taking RT on both sides, we have
$R_{z}\left[\operatorname{erf}\left(\frac{a}{2 \sqrt{\mathrm{z}}}\right)\right]=\frac{2}{\sqrt{\pi}} \mathrm{r}^{3} \int_{0}^{\infty} e^{-\mathrm{rz}} \int_{0}^{\frac{a}{2 \sqrt{\mathrm{z}}}} e^{-x^{2}} \mathrm{dx} d z$
Or
$R_{Z}\left[\operatorname{erf}\left(\frac{a}{2 \sqrt{z}}\right)\right]=\frac{2}{\sqrt{\pi}} \mathrm{r}^{3} \int_{0}^{\infty} \int_{0}^{\frac{a}{2 \sqrt{z}}} e^{-\mathrm{rz}} e^{-x^{2}} \mathrm{dz} d x$
On trucking the sequence of integration and heeding that $0<\mathrm{x}<\frac{a}{2 \sqrt{\mathrm{z}}}$ and $0<\mathrm{z}<\infty$. Then $0<\mathrm{z}<\frac{a^{2}}{4 x^{2}}$ and $0<\mathrm{x}<$ $\infty$.
This yields
$R_{y}\left[\operatorname{erf}\left(\frac{a}{2 \sqrt{y}}\right)\right]=\frac{2}{\sqrt{\pi}} \mathrm{r}^{3} \int_{0}^{\infty} \int_{0}^{\frac{a^{2}}{4 x^{2}}} e^{-\mathrm{ry}} e^{-x^{2}} \mathrm{dy} d x$
Or
$R_{y}\left[\operatorname{erf}\left(\frac{a}{2 \sqrt{\mathrm{y}}}\right)\right]=\frac{2}{\sqrt{\pi}} \mathrm{r}^{2} \int_{0}^{\infty} e^{-x^{2}}\left[1-e^{-\frac{\mathrm{r} a^{2}}{4 x^{2}}}\right] d x$
Or

$$
\begin{aligned}
R_{z}\left[\operatorname{erf}\left(\frac{a}{2 \sqrt{\mathrm{z}}}\right)\right]= & \frac{2}{\sqrt{\pi}} \mathrm{r}^{2} \int_{0}^{\infty} e^{-x^{2}} d x \\
& -\frac{2}{\sqrt{\pi}} \mathrm{r}^{2} \int_{0}^{\infty} e^{-x^{2}} e^{-\frac{\mathrm{r} \frac{a^{2}}{4 x^{2}}}{} d x}
\end{aligned}
$$

Or
$R_{Z}\left[\operatorname{erf}\left(\frac{a}{2 \sqrt{\mathrm{Z}}}\right)\right]=\frac{2}{\sqrt{\pi}} \mathrm{r}^{2} \frac{\sqrt{\pi}}{2}-\frac{2}{\sqrt{\pi}} \mathrm{r}^{2} \int_{0}^{\infty} e^{-x^{2}-\frac{\mathrm{r} a^{2}}{4 x^{2}}} d x$
Or

$$
\begin{equation*}
R_{z}\left[\operatorname{erf}\left(\frac{a}{2 \sqrt{\mathrm{z}}}\right)\right]=\mathrm{r}^{2}-\frac{2}{\sqrt{\pi}} \mathrm{r}^{2} \int_{0}^{\infty} e^{-x^{2}-\frac{\mathrm{r} a^{2}}{4 x^{2}}} d x \tag{12}
\end{equation*}
$$

$$
\begin{equation*}
\text { Let } I(\mathrm{r})=\int_{0}^{\infty} e^{-x^{2}-\frac{\mathrm{r}}{} \mathrm{a}^{2}} \frac{x^{2}}{} d x \tag{13}
\end{equation*}
$$

Using the Leibniz instruction [16, 17], we find

$$
\begin{equation*}
I^{\prime}(\mathrm{r})=\int_{0}^{\infty} e^{-x^{2}-\frac{\mathrm{r}}{} a^{2}} 4 x^{2}\left(-\frac{a^{2}}{4 x^{2}}\right) d x \tag{14}
\end{equation*}
$$

Put $z^{2}=\frac{\mathrm{r} a^{2}}{4 x^{2}}, d x=-\frac{a \mathrm{r}^{\frac{1}{2}}}{2 z^{2}} d z$, and simplifying, we have
$I^{\prime}(\mathrm{r})=-\frac{a}{2 \sqrt{\mathrm{r}}} \int_{0}^{\infty} e^{-z^{2}-\frac{\mathrm{r} a^{2}}{4 z^{2}}} d z$
Using equation (13), we find
$I^{\prime}(\mathrm{r})=-\frac{a}{2 \sqrt{\mathrm{r}}} I(\mathrm{r})$
$I^{\prime}(\mathrm{r})+\frac{a}{2 \sqrt{\mathrm{r}}} I(\mathrm{r})=0$
Solving this equation, we find

$$
\begin{equation*}
I(\mathrm{r})=\mathrm{H} e^{-a \sqrt{\mathrm{r}}} \tag{15}
\end{equation*}
$$

To find H, a constant, from (13), we have
$I(0)=\int_{0}^{\infty} e^{-x^{2}} d x=\frac{\sqrt{\pi}}{2}$
Therefore, $\mathrm{H}=\frac{\sqrt{\pi}}{2}$.

$$
\begin{equation*}
\text { Hence } I(\mathrm{r})=\int_{0}^{\infty} e^{-x^{2}-\frac{\mathrm{r} a^{2}}{4 x^{2}}} d x=\frac{\sqrt{\pi}}{2} e^{-a \sqrt{r}} \tag{16}
\end{equation*}
$$

Using (16) in (12), we have
$R_{z}\left[\operatorname{erf}\left(\frac{a}{2 \sqrt{z}}\right)\right]=\mathrm{r}^{2}-\frac{2}{\sqrt{\pi}} \mathrm{r}^{2} \frac{\sqrt{\pi}}{2} e^{-a \sqrt{\mathrm{r}}}$
Or

$$
\begin{equation*}
R_{z}\left[\operatorname{erf}\left(\frac{a}{2 \sqrt{z}}\right)\right]=\mathrm{r}^{2}-\mathrm{r}^{2} e^{-a \sqrt{\mathrm{r}}} \tag{17}
\end{equation*}
$$

The RT $[12,13]$ of the complementary error function is given by
$R_{z}\left[\operatorname{erf} f_{c}\left(\frac{a}{2 \sqrt{\mathrm{Z}}}\right)\right]=R_{z}\left[1-\operatorname{erf}\left(\frac{a}{2 \sqrt{\mathrm{Z}}}\right)\right]$
Or
$R_{z}\left[\operatorname{erf} f_{c}\left(\frac{a}{2 \sqrt{\mathrm{Z}}}\right)\right]=R_{z}[1]-R_{z}\left[\operatorname{erf}\left(\frac{a}{2 \sqrt{\mathrm{Z}}}\right)\right]$
Or

$$
\begin{aligned}
& R_{Z}\left[\operatorname{erf} f_{c}\left(\frac{a}{2 \sqrt{\mathrm{Z}}}\right)\right]=R_{Z}\left[1-\operatorname{erf}\left(\frac{a}{2 \sqrt{\mathrm{Z}}}\right)\right] \\
& =r^{2}-\left(r^{2}-r^{2} e^{-a \sqrt{\mathrm{r}}}\right)=r^{2} e^{-a \sqrt{\mathrm{r}}} \\
& R_{z}\left[\operatorname{erf} f_{c}\left(\frac{a}{2 \sqrt{\mathrm{z}}}\right)\right]=R_{z}\left[1-\operatorname{erf}\left(\frac{a}{2 \sqrt{\mathrm{z}}}\right)\right]=r^{2} e^{-a \sqrt{\mathrm{r}}}
\end{aligned}
$$

Hence the inverse RT of [ $\left.r^{2} e^{-a \sqrt{r}}\right]$ is
$\left[1-\operatorname{erf}\left(\frac{a}{2 \sqrt{\mathrm{Z}}}\right)\right]=\operatorname{erf}_{c}\left(\frac{a}{2 \sqrt{\mathrm{Z}}}\right)$.
Hence equation (11) can be written as

$$
\begin{array}{r}
\mathrm{V}(\mathrm{z}, \mathrm{t})=\mathrm{V}_{0}\left[1-\operatorname{erf}\left(\frac{z \sqrt{\mathrm{CR}}}{2 \sqrt{\mathrm{t}}}\right)\right] \\
\mathrm{V}(\mathrm{z}, \mathrm{t})=\mathrm{V}_{0} e r f_{c}\left[\frac{z \sqrt{\mathrm{CR}}}{2 \sqrt{\mathrm{t}}}\right] \\
\mathrm{V}(\mathrm{z}, \mathrm{t})=\mathrm{V}_{0} \frac{z \sqrt{C R}}{2 \sqrt{\pi}} \int_{0}^{t} w^{-\frac{3}{2}} e^{-\frac{R C z^{2}}{4 w}} d w \tag{18}
\end{array}
$$

From (5), we have

$$
\begin{equation*}
I(\mathrm{z}, \mathrm{t})=-\frac{1}{R} \frac{\partial V(z, t)}{\partial z} \tag{19}
\end{equation*}
$$

Using (18) in (19), we get
$I(\mathrm{z}, \mathrm{t})=-\frac{1}{R} \frac{\partial}{\partial z}\left(\mathrm{~V}_{0} \frac{z \sqrt{\mathrm{RC}}}{2 \sqrt{\pi}} \int_{0}^{t} w^{-\frac{3}{2}} e^{-\frac{R C z^{2}}{4 w}} d w\right)$
Or
$I(\mathrm{z}, \mathrm{t})=-\frac{\mathrm{V}_{0}}{R} \frac{\partial}{\partial z}\left(\frac{z \sqrt{\mathrm{RC}}}{2 \sqrt{\pi}} \int_{0}^{t} w^{-\frac{3}{2}} e^{-\frac{R C Z^{2}}{4 w}} d w\right)$
On simplifying, we get

$$
\begin{equation*}
I(\mathrm{z}, \mathrm{t})=\frac{\mathrm{v}_{0}}{2 \sqrt{\pi}} \sqrt{\frac{C}{R}} t^{-\frac{3}{2}} z e^{-\frac{C R z^{2}}{4 t}} \tag{20}
\end{equation*}
$$

Taking $\mathrm{V}_{\mathrm{o}}=10000$ volt, $\mathrm{C}=1.0$ microfarad, $\mathrm{R}=10000$ ohm, the graph of $\mathrm{I}(\mathrm{z}, \mathrm{t})$ is shown in figure below:


If $\mathrm{z}=10 \mathrm{~m}$, then the graph between $\mathrm{I}(10, \mathrm{t})$ and t is shown in the figure below:


If $\mathrm{t}=100$ seconds, the graph between $\mathrm{I}(\mathrm{z}, 100)$ and z is shown in the figure below:

$$
\mathrm{I}(\mathrm{z}, 100)
$$



## 3. Results and Discussion

The double RT has been attempted favorably for examining the wave equations of transferral lines with negligible losses earth or deck through dielectric or nonconductor. With the ease of double RT exercise in dissimilar engineering areas, the other problems can also be examined easily.

It is clear from the graphs that for a fixed time $t$, current I first increase with an increase in length $z$ and then starts decreasing exponentially and then becomes zero. For a fixed length $z$, current I decrease with an increase in time $t$ and then becomes zero.

## 4. Conclusion

The double RT has come out to be a constructive tool for examining the wave equations of electrical transferral lines. The result procured, i.e. the elucidations of general equations of transferral lines with negligible losses earth or deck through dielectric or non-conductor, is the same as procured with other practices [1-3, 19-26].

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